Frontiers in Applied Dynamical Systems

University College Cork, Ireland, 18-20 June, 2024

Time	**Tuesday 18 June**	**Location: Aula Maxima**
8:45-9:30	Registration	
9:30-9:35	Workshop opening	
Chair: Andrew Keane		
9:35-10:05	Daan Lenstra	
10:10-10:40	Sebastian Wieczorek	
10:45-11:10	*Coffee/Tea break*	
Chair: Johan Dubbeldam		
11:10-11:40	Soizic Terrien	
11:45-12:15	James Knowles	
12:45-13:40	*Lunch in the Staff Restauarant (provided)*	
13:40-15:00	*Guided historical tour of UCC*	
Chair: Harry Dankowicz		
15:00-15:30	John Guckenheimer (Online)	
15:35-16:05	Sylvain Barbay	
16:10-17:30	*Discussions*	
18:00	*Welcome reception Franciscan Well Brewery/Pub*	

Time	**Wednesday 19 June**	**Location: Aula Maxima**
Chair: Jan Sieber		
9:30-10:00	Chris Budd	
10:05-11:05	Harry Dankowicz	
10:40-11:10	*Coffee/Tea break*	
Chair: Jen Creaser		
11:10-11:40	Jon Rubin	
11:45-12:15	John Terry	
12:45-14:00	*Lunch (provided)*	
14:00-15:00	*Discussions*	
Chair: Soizic Terrien		
15:00-15:30	Dana C'Julio	
15:35-16:05	Ale Jan Homburg	
16:15-16:45	Poster introductions	
17:00-19:00	Poster session, Irish music, drinks and snacks	

Time	**Thursday 20 June**	**Location: Aula Maxima**
Chair: Sebastian Wieczorek		
9:30-10:00	Rajarshi Roy	
10:05-10:35	Kathy Luedge	
10:40-11:10	*Coffee/Tea break*	
Chair: Jan Sieber		
11:10-11:40	Pablo Aguirre	
11:45-12:15	Tony Humphries	
12:45-14:00	*Lunch (provided)*	
14:00-15:00	*Discussions*	
15:00	*Excursion and workshop dinner*	

Titles and abstracts of oral presentations

Excitability in Eindhoven

Daan Lenstra, Lukas Puts and Weiming Yao, EHCI, Eindhoven, The Netherlands

In this talk I will overview the activities within the Eindhoven Hendrik Casimir Institute (EHCI) toward the realization of an integrated photonic device that mimics the operation of a spiking neuron in the brain. The central theme is the quest for excitability in a two-section semiconductor laser.

Rate-induced Tipping Points: A Tale of Invariant Manifolds

Sebastian Wieczorek, School of Mathematical Sciences, UCC, Cork, Ireland

Put simply, tipping points are large and sudden changes in the state of a complex system (e.g. Earth's climate, ecosystem, brain) in response to small and slow changes in external inputs. Consider a complex system near a stable state (an attractor), which we refer to as the base state. We might expect that, as external inputs change with time, the base state will change too. In many cases the system may adapt to changing external inputs and track the moving base state. However, tracking may not always be possible due to nonlinearities, competing timescales and feedbacks in the system.
In this talk, I will describe the interesting and relatively new phenomenon of rate-induced tipping (R-tipping). R-tipping occurs when the rate of change of the external input is in a certain interval, bounded by critical rate(s), where the system fails to adapt and track the moving base state, crosses some threshold and transitions to a different state. The different state may be long-lived (another attractor in a multi-stable system) or short-lived (a transient response in an excitable system). Crucially, R-tipping is a genuine nonautonomous instability that requires development of mathematical techniques beyond the classical autonomous bifurcation theory.
I will describe ideas, underpinned by a special compactification technique, that allow us to study R-tipping, find the critical rates and describe what happens when they are exceeded, in terms of intersecting invariant manifolds of saddles from infinity. I will illustrate these ideas with examples of natural systems, including R-tipping in PDE models of geographically shifting habitats with changing climate.

Merging and Disconnecting Resonance Tongues in an Excitable Laser with Delayed Feedback

Soizic Terrein, Le Mans University, France

Excitability is a general phenomenon encountered in fields from biology to neurosciences and optics, which manifests itself by an all-or-none response of a system to an external perturbation. Here we consider a microlaser with integrated saturable absorber. It displays an excitable response in the form of a short, high-amplitude light pulse when subject to an optical perturbation whose amplitude exceeds the so-called excitable threshold. In the presence of delayed feedback, this system has been shown to sustain multistable pulsing regimes with different numbers of regularly timed pulses. Recently, periodic pulsing regimes with non-equidistant pulses have been observed, which possibly coexist with equidistant pulsing patterns.
Here we perform a bifurcation analysis of a suitable mathematical model — a system of three delay-differential equations — in the space of feedback strength, feedback delay and pumping parameter. This unveils the key role played by resonance tongues in the emergence of complex dynamics. In particular, we show that non equidistant pulsing patterns exist in unexpectedly large disconnected locking regions of the parameter space. These result from a merging and disconnecting process of resonance tongues that implies successive saddle transitions of the saddle-node bifurcations that bound resonance tongues, as well as the interaction of resonance tongues at an extremum of the rotation number along a torus bifurcation curve. This general mechanism is illustrated in two generic cases of disconnecting and disappearing resonance tongues.

Application of Bifurcation Analysis in the Aerospace Sector

James Knowles, Loughborough University, UK

Engineers have traditionally used time history simulations to study nonlinear dynamics, which can be a time-consuming approach that often misses key dynamic features (such as unstable limit cycles). At the beginning of the millennium, Bernd and colleagues at The University of Bristol built a research collaboration with Airbus in the UK which ran for over a decade, culminating the Airbus adopting dynamical systems tools within their future projects department. This talk presents some research highlights from that collaboration, and looks at some activities that members of the self-titled "airbus research group" are involved with now.

A Devil's Staircase of Principal Foliations

John Guckenheimer, Cornell University, Ithaca, USA (Online)

Principal foliations consisting of lines of curvature on surfaces emdedded in three space are similar to vector fields on two dimensional manifolds, but they are not orientable. The principal foliations of triaxial ellipsoids were described by Monge at the end of the eighteenth century. This talk will describe principal foliations in a one parameter family that deforms an ellipsoid. The results are surprising: the surfaces have "return maps" that are piecewise smooth homeomorphisms of the circle. The surfaces have dense lines of curvature at a Cantor set of parameters, but unlike smooth diffeomorphisms of the circle, this Cantor set has Lebesgue measure zero.

Neuromorphic dynamics in excitable micropillar lasers

Sylvain Barbay, Université Paris-Saclay, France

Micropillar lasers with integrated saturable absorber show good promise for ultra-fast analog neuromorphic computing. They display the basic properties of biological neurons (excitability, refractory periods, latency, temporal summation) but with much faster timescales. When subject to delayed optical feedback, they form an autapse which is at the basis of long and short term memories. We also unveil a novel symmetry breaking phenomenon in the timings of excitable spikes. All these observations are in very good agreement with the theoretical analysis of a simple model.

Dynamic tipping and cyclic folds, in a one-dimensional non-smooth dynamical system linked to climate models

Chris Budd, University of Bath, UK

In this talk I will consider the behaviour at tipping points close to (smoothed) non-smooth fold bifurcations in one-dimensional oscillatory forced systems with slow parameter drift. The focus of the talk is the non-smooth fold in the Stommel 2-box, and related climate models, which are piecewise-smooth continuous dynamical systems, modelling thermohaline circulation. These exhibit non-smooth fold bifurcations which arise when a saddle-point and a focus meet at a border collision bifurcation. By using techniques from the theory of non-smooth dynamical systems I will provide precise estimates for the general tipping behaviour close to the non-smooth fold as parameters vary. These are different from the usual tipping point estimates at a saddle-node bifurcation, with advanced tipping apparent in the non-smooth case when compared to the behaviour near smoothed approximations to this fold. I will also show rapid, and non-monotone, transitions in the tipping points for oscillatory forced systems close to both non-smooth folds and saddle-node bifurcations due to the effects of both phase changes and non-smoothness. These variations can have implications for the prediction of tipping in climate systems, particularly in close proximity to a non-smooth fold.

Parameter Continuation and Uncertainty Quantification Near Stochastically Perturbed Limit Cycles and Tori

Harry Dankowicz, University of Maryland, USA

This talk shows the use of parameter continuation techniques to characterize intermediate-term dynamics due to the presence of small Brownian noise near normally-hyperbolic, transversally-stable periodic orbits and quasiperiodic invariant tori found in the deterministic limit. The proposed formulation relies on adjoint boundary-value problems for constructing continuous families of transversal hyperplanes that are invariant under the linearized deterministic flow, and covariance boundary-value problems for describing Gaussian distributions of intersections of stochastic trajectories with these hyperplanes. Analytical and numerical results, including validation with the help of the continuation package COCO, show excellent agreement with stochastic time integration for problems with either autonomous or time-periodic drift terms.

Timescale Separation and Control of Neuronal Bursting

Jonathan Rubin, University of Pittsburgh, USA

Much of the theoretical work on neuronal bursting has related to classifying burst patterns and linking them with bifurcation structure of fast dynamics within underlying models. In this talk, I will discuss ways that, within a given burst class, quantitative features can be adjusted or the parameter range over which bursting occurs can be expanded by exploiting the fast-slow decomposition approach. I will also talk about using related ideas to introduce new bursting patterns that resemble those seen in some neuronal systems, such as the mammalian respiratory pattern generator. This is joint work with various collaborators including some guy from Auckland named Krauskopf.

Dynamic Network Models for Diagnosis and Treatment of Epilepsy: From Waterfall Diagrams to a Medical Technology

John Terry, University of Birmingham, UK

In this talk I recall how an expert in dynamical systems, especially those with delay in the context of lasers, was introduced to the brain. Despite promising never to branch into medicine I describe how his knowledge was utilised in the context of understanding the response of deep brain nuclei to stimulation and its potential as a therapeutic treatment for people with epilepsy. From there, I describe more recent work where dynamical systems on networks (with or without delay) have been utilised to understand large-scale mechanisms by which the brain can transition to seizure. I explore how this understanding can be utilised in the context of diagnosis and prognosis of epilepsy and outline the journey of translation that has seen fundamental mathematical research result in a successful startup company.

Computational Tool to Identify the Emergence of Blenders in a Three-dimensional Hénon-like Map

Dana C'Julio, University of Auckland, New Zealand

A dynamical system given by a diffeomorphism with a three-dimensional space may have a *blender*, which is a hyperbolic set Λ with, say, a one-dimensional stable invariant manifold that behaves like a surface. This means that the stable manifold of any fixed or periodic point in Λ weaves back and forth as a curve in phase space such that it is dense in some planar projection; we refer to this as the *carpet property*. Moreover, this property persists for small arbitrary perturbations. The presence of a blender can be used to prove the robust existence of a non-transverse heterodimensional cycle, which implies the so-called wild chaos.

We present an algorithm for computing very long pieces of such a one-dimensional manifold that we implemented in MATLAB to identify efficiently, accurately and reliably a very large number of intersection points with a specified section. With this method, we investigate the geometric properties of the emergence of blenders in a three-dimensional Hénon-like map $\mathcal{H}$ by computing the one-dimensional manifold of its fixed and periodic points. Specifically, we discover that the organisation of the projection of the intersection points follows a consistent pattern as a parameter is varied. Consequently, we can study in detail the specific parameter values and order of events that occur as the blender arises.

Why Study Iterated Function Systems

Ale Jan Homburg, University of Amsterdam, The Netherlands

I will discuss dynamical phenomena such as synchronization by noise and intermittency, in the simplest possible contexts of iterated functions systems. The settings include for instance iterated function systems generated by piecewise affine contracting and expanding maps on the unit interval.

प्राण बन्धु (What does this mean?)

Rajarshi Roy, University of Maryland, College Park, USA

Randomness and determinism are entwined in our lives and impact how every day evolves. This talk will reflect on these aspects of nature as illustrated through studies of light and matter. Particles and waves, coherence and incoherence, will be illustrated through examples in science and engineering that have profoundly affected how we live and communicate with electromagnetic waves/photons. We will conclude with a brief discussion of the question - how much information can a single photon communicate?

Photonics Reservoir Computing: A Use Case for Bifurcation Theory

Kathy Lüdge, Technische Universität Ilmenau, Germany

Optical laser cavities with delayed-coupling are widely explored candidates for photonic reservoir computing (RC). The latter is a machine learning scheme where only one linear regression step at the readout layer is needed while the reservoir itself remains unchanged. RC can thus be easily implemented in various hardware systems. It is especially suited for solving complex time-series prediction tasks due to the memory provided by the finite internal timescales of the dynamical system used as the reservoir. The specific RC prediction performance, however, critically depends on the underlying bifurcation structure of the nonlinear dynamical system used as the reservoir.
In this talk, I will present ways to improve the performance of delay-based RC systems via timescale tuning and bifurcation analysis. I will also discuss possibilities for external memory augmentation via additional delays, which can greatly improve the performance and reduce the parameter dependencies of the RC system.

Global Invariant Manifolds and Their Applications

Pablo Aguirre, Universidad Técnica Federico Santa María, Chile

One of the most celebrated aspects of the work of Prof Bernd Krauskopf has been the development of new techniques to calculate, analyse and visualise global invariant manifolds of vector fields and maps. In this talk I will give a brief overview of my own involvement in this area and the impact Prof Krauskopf has had on my work from my early days in Bristol up to the present.

Dynamics of State-Dependent Delay Differential Equations

Tony Humphries, McGill University, Montreal, Canada

Delay differential equations (DDEs) define infinite dimensional dynamical systems, and the infinite dimensionality can lead to very interesting dynamics even in simple looking equations. We consider a model equation with one or two delays which has no nonlinearity except for state-dependency of the delays, which are linearly state-dependent. The state-dependency of the delays ensures that the dynamics remain bounded even when the trivial steady-state solution becomes unstable. We describe initial and boundary value computations of the resulting dynamics, and Hopf bifurcations and saddle node and period-doubling bifurcations of periodic orbits. Double-Hopf bifurcations give rise to invariant tori, including at least one stable torus, that we investigate in detail. We also briefly consider the dynamics of the DDE in a singularly perturbed limit and with a delayed delay.
