A Scalable Uncertainty Quantification Framework for Nonlinear Dynamical Systems

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Uncertainty is extensively seen in dynamical systems and can lead to unanticipated behaviours that trigger damage and catastrophe. One important aspect of uncertainty quantification involves identifying the envelope that bounds the uncertain behaviours. However, this task is typically computationally expensive, especially in the presence of many uncertain sources and complex nonlinear behaviours. This poster proposes a scalable uncertainty quantification framework for nonlinear dynamical systems showing periodic responses. Firstly, the envelope-identification problem is interpreted as a propagation problem (in the sense of numerical continuation) — the uncertain envelope is grown from the reference condition (without uncertainty) with increasing uncertainty. The direction of propagation resides in a 1-dimensional subspace of the uncertain parameter domain, enabling the scalability for problems with high-dimensional uncertainty. This imposes strong assumptions on the underlying dynamical system, but these assumptions appear to hold in the examples tested. The propagation problem, comprising the envelope conditions and the definition of the periodic orbit, is then constructed and implemented using a path-following algorithm. Examples are presented to show the accuracy of the proposed method as well as its capability in capturing complex nonlinear scenarios, including modal interactions and the emergence of an isola.

A cancer model with varying extinction threshold reproduces breast cancer data

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We propose a conceptual model of cancer development that captures both carcinogenesis and subsequent cancer progression. One of the central concepts of the model is an extinction threshold akin to the strong Allee effect in population biology. First, we uncover limitations of the commonly used strong Allee effect model, initially proposed by Vito Volterra, to reproduce typical cancer progression. Addressing these limitations, we propose a new cancer progression model with different forms of cancer progression above the extinction threshold ranging from logistic to Gompertzian growth. Second, we propose a cancer development model that incorporates three processes: (i) random mutations of stem cells at a rate that can increase over time due to ageing, (ii) cancer suppression mechanisms whose strength can also change over time, giving rise to a time-varying extinction threshold, and (iii) our new proposed progression model. Using tuned parameters and breast cancer as a case study, we show that our model reproduces typical cancer progression, generates testable predictions, and gives new insight into observed non-trivial cumulative lifetime risk for breast cancer.

Arnold tongues in the brain

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Deep brain stimulation is a treatment for Parkinson's disease, involving implantation of electrodes into patient's brain, and continuously providing pulsatile stimulation with frequency around 130 Hz. It has been recently observed that the stimulation at high frequencies (e.g., 130Hz) may entrain and amplify oscillations with half frequency (e.g., 65Hz) that are related with movements. Such entrainment may cause side-effects of the treatment such as movements of hands against the will of the patient. In this work, simple mathematical models have been developed to describe the entrainment of neural oscillations by the stimulation. The models predict the existence of entrainment regions in stimulation parameter space, known as the Arnold tongues. The existence of such tongues has been subsequently verified in an experiment in which the frequency and amplitude of stimulation provided to patients have been varied systematically. It has been also investigated how the stimulation could be adjusted to avoid the undesired entrainment.

Geometry of wild chaos: the impact of the orientation properties of 3D horseshoes for blenders

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Wild chaos is a new type of dynamics with certain robustness properties that can arise in diffeomorphisms of dimension at least three. An important ingredient in this context is a blender: a hyperbolic set with invariant manifolds that seem to be higher dimensional than expected. The existence of a blender implies the robust presence of complicated global connections between different parts of phase space.

What does a blender actually look like, and how does it appear or disappear? We consider a Hénon-like family of maps that exhibits a blender generated by a three-dimensional horseshoe. We investigate how the orientation properties of the map affect the structure of the blender. To this end, we employ advanced numerical techniques to compute very long pieces of the one-dimensional stable manifolds of fixed points in a compactified phase space. This allows us to characterise how blenders lose their defining property, namely, their one-dimensional manifolds no longer behave like surfaces.

Geometric Models of Wild Chaos

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Wild chaos is a new type of chaotic dynamics that can only arise in systems of sufficiently high dimension. It is characterised by the robust presence of non-hyperbolic dynamics; in particular, homoclinic and/or heteroclinic tangencies occur robustly in open parameter intervals. We take a geometric approach to study the existence and nature of this type of chaos and search for defining features in the skeleton structure of invariant dynamics formed by fixed points, periodic orbits, and associated stable and unstable manifolds. One of the fascinating markers of wild chaos is that some of the invariant manifolds in the system exhibit geometric properties that are only typical for manifolds of a higher dimension. We investigate the presence of wild chaos in a three-dimensional discrete-time system generated by the so-called shear-rotation map. This is one of the very few examples of systems that may exhibit wild chaos. Stable and unstable manifolds in this system appear to form a robust heteroclinic tangle, which generates a transitive hyperbolic set that exhibits chaotic dynamics, despite the fact that both manifolds are only one dimensional. We compute these one-dimensional invariant manifolds and observe their perceived intersection set; for comparison, we also approximate the underlying chaotic set by computing a very large number of periodic orbits.

Transitions to wild chaos in a 4D Lorenz-like system

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Wild chaos is a form of higher-dimensional chaotic dynamics that can only arise in vector fields of dimension at least four. We study wild chaos in a four-dimensional system of differential equations, which is a four-parameter extension of the classic Lorenz equations. Recently, Gonchenko, Kazakov and Turaev (2021) showed, via the computation of Lyapunov exponents, that this system has a wild chaotic attractor at a particular point in parameter space. We investigate how this wild chaotic attractor arises geometrically, performing a bifurcation analysis of the system in a two-parameter setting. As a starting point, we continue the one-parameter bifurcation structure of the classic Lorenz equations when the fourth parameter is "switched on". We find that the well-known homoclinic explosion point of the Lorenz system unfolds and gives rise to infinite cascades of global connections in the four-dimensional system that are of Shilnikov type. These connections are formed by the unstable manifold of the origin, which still plays an essential role in the emergence of complicated dynamics in the system. We also compute the kneading diagram that encodes how this one-dimensional manifold repeatedly moves around a pair of equilibria. In combination with the direct computation of curves of global bifurcations, the kneading diagram provides insight that helps identify regions where wild chaos may occur.

Multi-objective optimisation for sustainable transitions: A conceptual framework for system transformation

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Doughnut Economics is a theoretical framework inspired by the UN's Sustainable Development Goals and the concept of Planetary Boundaries, and designed to help establish a safe and just space for humanity to thrive. Using this framework, we identify a multitude of social and environmental factors that are implicated in ensuring a sustainable system transformation. Each factor is associated with one or more targets that can be realised through strategic policy-making. Two primary barriers make this difficult to achieve: (1) targets are often in conflict with each other, and (2) policies and interventions are commonly planned in isolation, not accounting for complex interdependencies, which can lead to negative unintended consequences. Thus, a holistic approach to system transformation becomes imperative. To address this, we propose a general analytical framework that predicts the impact of policies and decision-making on targets and outcomes while capturing the intricate system interdependencies in a policy-target network. Inspired by Kauffman's NK fitness landscape, our framework takes the form of a multi-objective optimisation model that employs a dynamical evolutionary algorithm in conjunction with network analysis. Our algorithm accounts for tradeoffs between conflicting targets in the network by dynamically reallocating resources from inadequate policies to those that are more efficient and impactful. One key finding is that a short-term disruption for a subset of targets is necessary to reach optimal outcomes in the long run. Simulation-based sensitivity analysis demonstrates that the quality and nature of policies are paramount, rather than their sheer quantity. We also identify an optimal level of budgeting that efficiently minimises the compromise between performance and delivery time. This study serves a foundational step towards an interactive tool that assists stakeholders and decision-makers in identifying, prioritising, and resourcing policies for achieving optimal outcomes for problems with a large number of interacting targets.

Nonlinear dynamics in automotive engineering: a study of tyre friction

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Tyre friction allows cars to accelerate, brake and corner, but it can also induce limit cycle oscillations. This work presents two nonlinear phenomena that occur in cars because of the friction force generated by the tyres: specifically, wheel hop and steering shimmy. The existence and locations of Hopf bifurcations are shown to be influenced by tyre pressure, but this relation changes depending on how the tyre model captures the tyre's friction force dependency on pressure. Being able to predict this dependency is an open scientific problem, with the interaction between rubber and the road surface a significant yet currently unpredictable factor that this work is currently addressing.

Bifurcation analysis of a two-delay model for the Atlantic Meridional Overturning Circulation

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We consider a conceptual model for the Atlantic Meridional Overturning Circulation (AMOC) in the form of a scalar delay differential equation (DDE) with two time delays. The time delays are associated with the negative salinity feedback between the Equator and the North Pole, and the density exchange between the surface and deep water near the Pole. After rescaling, the delays are the only parameters of the model. The presented model is interesting beyond the context of the AMOC as a new type of DDE with two multiplicative delay terms, which is effectively the logistic growth equation with two independent delay times. As such, this DDE can be seen as a generalization of the Hutchinson-Wright equation, which has been introduced in two seminal papers in the rather different contexts of the distribution of primes and population dynamics. We perform a comprehensive bifurcation study of this DDE model as a function of the two delays. In particular, the system exhibits different types of complex oscillatory behavior, which we analyze with the software package DDE-Biftool for Matlab. This enables the identification and characterization of fascinating dynamics, including codimension-two Belyakov and Shilnikov-Hopf global bifurcations in the parameter plane of the two delays, which act as organizing centers for nearby dynamics. In particular, we discover previously unknown limiting periodic oscillations with rational ratios between the delays and the associated period, which we refer to as locking orbits. Moreover, we show where in the parameter space, nontrivial oscillations are stable and, hence, observable in the context of the AMOC. We observe that these stable solutions are organized in a repeating structure involving dynamics on invariant tori and associated bifurcations, which include torus break-up to chaotic behavior.